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ABSTRACT

The first two parts of this paper deal with the potential and reality of mathematics in education. In the third section, some recommendations are made for narrowing the gap which seems to exist between the potential and the reality. Among these fifteen recommendations are: (1) the public should be educated about the nature of mathematics; (2) steps must be taken to improve the attitude of mathematics teachers; (3) there should be a curriculum emphasis on the dynamic process of mathematizing rather than on the static product of this process which is mathematical fact; (4) we must stress the historical roots and contemporary applications of mathematics; and (5) the mathematical community must give educational considerations a higher priority than is now the case. (MK)

# REACH AND GRASP: OBSERVATIONS ON THE POTENTIAL AND REALITY OF MATHEMATICS EDUCATION

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# REACH AND GRASP: OBSERVATIONS ON THE POTENTIAL AND REALITY OF MATHEMATICS EDUCATION

William Higginson

*Ah, but a man's reach should exceed his grasp,*
*Or what's a heaven for?*

*R. Browning 'Andrea del Sarto'*

Like Caesar's Gaul, this paper is in three parts. The first two deal with the potential and the reality of mathematics in education. In the third section some recommendations are made for narrowing the gap which seems to exist between the potential and the reality.

The first question to be addressed is that of the role of mathematics in any comprehensive programme of education. What arguments can be made for including this discipline in a school or university curriculum? It will not come as a surprise, I think, given my vested interests, if I claim that because of its nature, mathematics should play a central role in any general programme of education.

There are two main arguments with which one might attempt to substantiate this claim. The first, the banausic or utilitarian argument, is orthodox and well-known. Briefly it runs: 'mathematics, like cod liver oil, is good for you; try it; you may not like it but, despite this, rest assured that it is doing good things for you'. In one of its primitive forms, this argument often has a strong hierarchical ring: you must study algebra because without algebra you can't do calculus and without calculus you can't become an engineer; thus algebra (or whatever) must be part of the total curriculum.

A higher order version of the banausic argument goes: our culture is characterized to a great extent by the available range and power of our technology; this technology is a direct product of scientific research and scientific research in turn rests completely on mathematics. Ergo, to understand the culture, one must understand the nature of its scientific foundations and this one can do only if one is mathematically knowledgeable.

A sophisticated version of this argument is given by the gifted contemporary critic of English literature, George Steiner, in his book In Bluebeard's Castle (1971):

> As in the twilit times of Ovid's fables of mutant being, we are in metamorphosis. To be ignorant of these scientific and technological phenomena, to be indifferent to their effects on our mental and physical experience, *is to opt out of reason*. A view of post-classic civilization must, increasingly, imply a vision of the sciences, of the language-worlds of mathematical and symbolic notation.
>
> It is often objected that the layman cannot share in the life of the sciences. He is 'bound to remain for ever a dumbfounded savage' before a world whose primary idiom he cannot grasp. Though good scientists themselves rarely say this, it is obviously true. But only to a degree. Modern science is centrally mathematical; the development of rigorous mathematical formalization marks the evolution of a given discipline, such as biology to full scientific maturity. Having no mathematics, or very little, the 'common reader' is excluded. If he tries to penetrate the meaning of a scientific argument, he will probably get it muddled, or misconstrue metaphor to signify the actual process. True again, but of a truth that is half-way to indolence. Even a modest mathematical culture will allow some approach to what is going on. The notion that one can exercise a rational literacy in the latter part of the twentieth century without a knowledge of calculus, without some preliminary access to topology or algebraic analysis, will soon seem a bizarre archaism. These styles and speech-forms from the grammar of number are already indispensable to many branches of modern logic, philosophy, linguistics and psychology. They are the language of feeling where it is, today, most adventurous. As electronic data-processing and coding pervade more and more of the economics and social order of our lives, the mathematical illiterate will find himself cut off. A new hierarchy of menial service and stunted opportunity may develop among those whose resources continue to be purely verbal. There may be 'word-helots'.
>
> (98-100)

The second argument I would raise to substantiate my claim that mathematics has great potential as an element in general education is, in my opinion, more important than the banausic one although it is much less orthodox. It is the mathetic argument, and it rests on the assumption that *homo sapiens* is, in some fundamental sense, <u>the</u> mathematical animal.

Now the question of the nature of human nature is a well-known intellectual quagmire and not an area one wishes to plunge into in a brief paper. (Even Plato ran into trouble on this question. One day in the Academy he defined man as a "featherless biped". One of his students named Diogenes went home and got himself a rooster which he proceeded to pluck. The next day he entered with the naked rooster, threw it into the circle and said, "Behold! Plato's man.")

The mathetic view is implicit in many of the Platonic dialogues (1961) but perhaps emerges most clearly in the writings of George Boole. Bertrand Russell wrote that pure mathematics began in 1854 with the publication of Boole's Laws of Thought. In this book Boole states:

> The laws of thought, in all its processes of conception and of reasoning, in all those operations of which language is the expression or the instrument, are of the same kind as are the laws of the acknowledged processes of Mathematics. It is not contended that it is necessary for us to acquaint ourselves with those laws in order to think coherently, or, in the ordinary sense of the terms, to reason well. Men draw inferences without any consciousness of those elements upon which the entire procedure depends. Still less is it desired to exalt the reasoning faculty over the faculties of observation, of reflection, and of judgment. But upon the very ground that human thought, traced to its ultimate elements, reveals itself in mathematical forms, we have a presumption that the mathematical sciences occupy, by the constitution of our nature, a fundamental place in human knowledge, and that no system of mental culture can be complete or fundamental, which altogether neglects them.
>
> (422-423)

The twentieth-century philosopher who perhaps came closest to the mathetic view was Ernst Cassirer (1953) with his idea of man as *animal symbolicum*.

Recent writings which seem consistent with the mathetic view, one corollary of which, for instance, would be that language acquisition is essentially a mathematical process, would include some of the work of Piaget, Shannon, Bronowski, Polya, Chomsky, and Spencer Brown. The latter, in his book Laws of Form (1973) writes:

> The discipline of mathematics is seen to be a way, powerful in comparison with others, of revealing our internal knowledge of the structure of the world, and only by the way associated with our common ability to reason and to compute.
>
> (xv)

Historically, variations on the mathetic theme can be seen in the writing of Leibniz, Pascal, Pythagoras, Descartes, Whitehead, Weyl, Korzybski, Spengler, and Gauss. The fact that infant prodigies are exclusively a phenomenon of mathematics and its two sister areas, chess and music, may be taken as another indication of the validity of the mathetic argument. The curious cases of the *idiots savants* (Ball, 1974) may be another. If it is true that the human animal is in some way 'wired mathematically' then surely it is consistent to contend that mathematics should constitute an important aspect of any scheme of general humanistic education.

But what of the reality of mathematics education? If, as has been contended, mathematics can claim to have considerable educational potential, how well is this potential being realized? There are many indications that the gap between potential and reality is very large, and that far from being a vehicle for human liberation and a means to an appreciation of truth and beauty, mathematics for many people has been an instrument of repression and fear.

The public image of mathematics has never been particularly good. In the seventh century, St. Augustine felt compelled to say, "The good Christian should beware of mathematicians and all those who make empty prophecies. The danger already exists that the mathematicians have made a covenant with the Devil to darken the spirit and confine Man in the bonds of Hell!" (Kline, 1954, p.3). Rousseau in his Confessions (1970) tells of an evening in Vienna spent in the company of a lady of less than spectacular virtue. We are spared details of the evening itself - (it appears that behavioral objectives were applied in those days as well) - but the next day the lady in question gave Jean-Jacques the following advice - "Give up the ladies and take up mathematics." (p.302). Stephen Leacock, the Canadian humourist (in this context it is also worth noting that Leacock was not a northern version of Will Rogers but rather a distinguished academic with a doctorate from the University of Chicago who was for many years head of the Economics Department at McGill University) reflected one time on his mathematical education, and he said as follows: "My attitude towards mathematics indeed is that of nine out of ten educated people - a sense of awe, something like horror, a gratitude for escape, but at times a wistful feeling of regret, a sense that there might have been a little more made of it".

Mathematicians recognize the public attitude towards their discipline as a social hazard. Many of us who at cocktail parties have confessed to being mathematicians by profession have been subjected to horrendous tales of personal ineptitude which, I suspect, are not elicited by any other area. Paul Halmos (1968), a distinguished contemporary mathematician, has written that on social occasions like this, or when travelling on aeroplanes, he's very tempted to tell people he's "in roofing and siding" (p.375).

How, then does this come about? The Mona Lisa is not used as a weapon. Why should an aesthetically appealing - (Whitehead (1948) called it "the most original creation of the human spirit" (p.25)) - construction like mathematics have become an object of fear, suspicion, and distaste in the minds of many able people? I think this is a specific illustration of the 'Mark Twain cat syndrome'. Anticipating the work of Professor Skinner by a number of years, this cat had absorbed its lessons well, for the unfortunate beast had once sat on a hot stove. Twain observed the cat, being an intelligent beast, never again sat on a hot stove but, on the other hand, it never sat on a cold one either.

The answer is to be found, in my opinion, in the way in which these people encounter the discipline, that is, in the way in which it is taught. Let me try to illustrate this point by two quotations. The first comes from Surprised by Joy (1959), the autobiography of C.S. Lewis (1898-1963) who, although a professor of English Literature at Oxford, is best remembered for his religious books such as The Screwtape Letters.

In the second chapter of Surprised by Joy, Lewis describes his school days and we're given a foretaste of what is to come by the title and the epigraph of the chapter which are, respectively, "Concentration Camp" and "Arithmetic with Coloured Rods". The passage is, so to speak, a striking one. He is writing about a school experience he had as a child of the age of twelve and one cannot help but be grateful that we have progressed so far, in some ways, in the last sixty-five years. It does not seem likely that this man who has influenced in a minor way the shape of our contemporary world, could avoid strong negative feelings in association with mathematics. We enter Surprised by Joy at the point where Lewis recalls the headmaster of his prep school, Oldie, and P, a fellow pupil who has made a mistake in a geometrical proof.

> I have seen Oldie make that child bend down at one end of the schoolroom and then take a run of the room's length at each stroke; but P. was the trained sufferer of countless thrashings and no sound escaped him until, towards the end of the torture, there came a noise quite unlike a human utterance. That peculiar croaking or rattling cry, that, and the grey faces of all the other boys, and their death-like stillness, are among the memories I could willingly dispense with.
>
> The curious thing is that despite all this cruelty we did surprisingly little work. This may have been partly because the cruelty was irrational and unpredictable; but it was partly because of the curious methods employed. Except at geometry (which he really liked) it might be said that Oldie did not teach at all. He called his class up and asked questions. When the replies were unsatisfactory he said in a low, calm voice, "Bring me my cane. I see I shall need it." If a boy became confused Oldie flogged the desk, shouting in a crescendo, "Think - Think - THINK!!" Then, as the prelude to execution, he muttered, "Come out, come out, come out." When really angry he proceeded to antics; worming for wax in his ear with his little finger and babbling, "Aye, aye, aye, aye...." I have seen him leap up and dance round and round like a performing bear. Meanwhile, almost in whispers, Wee Wee or the usher, or (later) Oldie's youngest daughter, was questioning us juniors at another desk. "Lessons" of this sort did not take very long; what was to be done with the boys for the rest of the time? Oldie had decided that they could, with least trouble to himself, be made to do arithmetic. Accordingly, when you entered school at nine o'clock you took up your slate and began doing sums. Presently you were called up to "say a lesson." When that was finished you went back to your place and did more sums - and so for ever. All the other arts and sciences thus appeared as islands (mostly rocky and dangerous islands)
>
> *Which like to rich and various gems inlaid*
> *The unadorned bosom of the deep*
>
> - the deep being a shoreless ocean of arithmetic. At the end of the morning you had to say how many sums you had done; and it was not quite safe to lie. But supervision was slack and very little assistance was given. My brother - I have told you that he was already a man of the world - soon found the proper solution. He announced every morning with perfect truth that he had done five sums; he did not add that they were the same five every day. It would be interesting to know how many thousand times he did them.
>
> (28-29)

If this case was an aberration it would perhaps be no cause for concern. Unfortunately, autobiographical literature reveals a high proportion of experiences like this. Jung, for instance, in Memories, Dreams, Reflections (1965) is another good example. Robertson Davies in World of Wonders (1975) has one of his characters muse, "Do you suppose the devil invented numbers?" (p.60). (Perhaps an analyst - Jungian or otherwise - might connect this statement to the fact that Davies dropped out of Queen's University in 1935, unable to pass first year mathematics.) W.H. Auden, in one of his briefest and least well-known poetic efforts, gives us some insight into his mathematical education:

*Minus times minus equals plus.*
*The reason for this we need not discuss.*

My second illustration is taken from more recent times, a passage from a book by the late American anthropologist, Jules Henry (1963). Instead of haring off to Brazil to study the exotic natives, he stayed at home in St. Louis for a few years and studied the exoteric natives there. The result of that study was a book called Culture Against Man which has a chapter called "Golden Rule Days: American Schoolrooms". In this chapter there is a scenario starring an unfortunate young man named Boris who is also encouraged to "think". I suggest that many of us can identify to a certain extent with him. Henry has some powerful observations to introduce and comment on the scenario.

> In a society where competition for the basic cultural goods is a pivot of action, people cannot be taught to love one another, for those who do cannot compete with one another, except in play. It thus becomes necessary for the school, without appearing to do so, to teach children how to hate, without appearing to do so, for our culture cannot tolerate the idea that babes should hate each other. How does the school accomplish this ambiguity? Obviously through competition itself, for what has greater potential for creating hostility than competition? One might say that this is one of the most "creative" features of school. Let us consider an incident from a fifth-grade arithmetic lesson.

*At the Blackboard*

> Boris had trouble reducing "12/16" to the lowest terms, and could only get as far as "6/8". The teacher asked him quietly if that was as far as he could reduce it. She suggested he "think". Much heaving up and down and waving of hands by the other children, all frantic to correct him. Boris pretty unhappy, probably mentally paralyzed. The teacher, quiet, patient, ignores the others and concentrates with look and voice on Boris. She says, "Is there a bigger number than two you can divide into the two parts of the fraction?" After a minute or two, she becomes more urgent, but there is no response from Boris. She then turns to the class and says, "Well, who can tell Boris what the number is?" A forest of hands appears, and the teacher calls Peggy. Peggy says that four may be divided into the numerator and the denominator.

Thus Boris' failure has made it possible for Peggy to succeed; his depression is the price of her exhilaration; his misery the occasion for her rejoicing. This is the standard condition of the American elementary school, and is why so many of us feel a contraction of the heart even if someone we never knew succeeds merely at garnering plankton in the Thames: because so often somebody's success has been bought at the cost of our failure. To a Zũni, Hopi, or Dakota Indian, Peggy's performance would seem cruel beyond belief, for competition, the wringing of success from somebody's failure, is a form of torture foreign to those noncompetitive redskins. Yet Peggy's action seems natural to us; and so it is. How else would you run our world? And since all but the brightest children have the constant experience that others succeed at their expense they cannot but develop an inherent tendency to hate - to hate the success of others, to hate others who are successful, and to be determined to prevent it. Along with this, naturally, goes the hope that others will fail. This hatred masquerades under the euphemistic name of "envy".

Looked at from Boris' point of view, the nightmare at the blackboard was, perhaps, a lesson in controlling himself so that he would not fly shrieking from the room under the enormous public pressure. Such experiences imprint on the mind of every man in our culture the *Dream of Failure*, so that over and over again, night in, night out, even at the pinnacle of success, a man will dream not of success, but of failure. *The external nightmare is internalized for life.* It is this dream that, above all other things, provides the fierce human energy required by technological drivenness. It was not so much that Boris was learning arithmetic, but that he was learning the *essential nightmare. To be successful in our culture one must learn to dream of failure.*

(295-296)

If it is true, as I have contended, that for mathematics as an element of education the gap between the potential and the reality is very large, what can be done to narrow this gap and hence make mathematics a more satisfying and worthwhile curriculum experience for students in the future?

Since space is short, I will merely list fifteen steps which, in my opinion, would alleviate the situation. Although many of the points refer specifically to mathematics education, several transcend subject boundaries and are, in my opinion, appropriate for other disciplines as well. While I have nct consciously played the devil's advocate, I do believe, with Sir Karl Popper, that the purpose of a paper is to provoke and hence have made no attempt to be other than blunt here.

1) We must educate the public about the nature of mathematics; the widely held view that mathematics is the science of number and space should be updated (Whitehead, 1948b) perhaps to the Bourbaki (1971) view that mathematics is 'the science of structure' (Weissinger, 1969) to more accurately catch the flavour of contemporary mathematics. (In this sense Freud was perhaps ahead of his time in being able to conceptualize different types of mathematics. He spoke at one time of the strange arithmetic of women; that one is too many and many is not enough.)

2) This is particularly true in the case of teachers, that is, the necessity of changing their vision of mathematics. Here, however, the question of attitude is central (Beltzner, 1976). Steps must be taken to improve the attitude of mathematics teachers, especially at the elementary school level, toward the subject. For many situations in the elementary school, the conception of mathematics as the science of pattern will be quite adequate (Sawyer, 1955; Whitehead, 1948c).

3) We must attempt to change teachers' perception of their own role away from the image of dispenser of information in the direction of the teacher as a model of an active learner (Higginson, 1973); this change should stress the importance of personal and professional growth.

4) This change in teacher role will be consistent with a curriculum emphasis on the dynamic process of mathematizing (Wheeler, 1974; Mason, 1976) rather than on the static product of this process which is mathematical fact. Einstein once made a distinction between science, the finished

product, a cold inhuman thing, and something very human, on the other hand, which he called 'science coming into being' (Holton, 1973, p.212). I think we have a very strong case for making mathematics coming into being part of our mathematics curricula (Polya, 1962, 1968; Lakatos, 1976).

5) We must stress, much more than we presently do, the historical roots (Menninger, 1969; Eves, 1976; Bronowski, 1973) and the contemporary applications (Holt, 1973); Science Council, 1974; Stevens, 1974) of mathematics; in particular we should emphasize the relationships between mathematics and the physical sciences (Hoyle, 1977; Polya, 1977; Kline, 1969).

6) The aesthetic and philosophical aspects of the subject (Heisenberg, 1972; Fuller, 1975; Critchlow, 1976; Capra, 1975) should be given much more prominence. We should, as well, stress mathematics as an ecologically sound recreation. When the 'cumulative horsepower equals status' era comes to an end - as it soon will - mathematics offers a clean, inexpensive, and non-polluting high.

7) The range and type of evaluation in mathematics should be considerably expanded (Association of Teachers of Mathematics, 1968).

8) Helping students learn 'how to think' should be seen as the major curriculum goal in mathematics; this will imply an emphasis on mathematical problem-solving (Polya, 1957, 1962).

9) The mathematical community must give educational considerations a higher priority than is now the case (Beltzner, 1976).

10) We need to carefully examine the implications and constraints of psychological and philosophical models on which mathematics curricula might be based; this is especially true, for example, in the areas of cognitive development, individualized instruction, and remediation. Neo-behaviourism, for example, should be seen as an inadequate theoretical base for all but the most trivial aspects of mathematics (Piaget, 1971; Kaufman, 1973; Bertalanffy, 1967; Koestler, 1970; Educational Technology, 1972).

11) We need to reject research paradigms and methodological techniques coming from other areas which are clearly inappropriate; the management version of teacher accountability and the very great percentage of educational research models fall into this category. For decades education has been the research design equivalent of clapped-out autos for Argentina. Just as the patched-up Packards and Corvairs of the world chug along the streets of Buenos Aires today, in educational research third-hand models with scientific pretension sputter along producing nothing significantly different in still another comparative study. Flow charting did not win the war in south-east Asia for the Pentagon and it cannot be seen as an appropriate technique for education in any true sense of the word. An immediate corollary of this is that we need to generate our own models and techniques (Educational Technology, 1973; Piaget, 1976; Hilton, 1975; Ginsburg, 1977, Krutetskii, 1976; Romberg, 1975).

12) The view that 'knowledge is power' (by Archimedes out of the Rand Corporation) leading to things like megadeath estimates, needs to be supplemented by the neo-Pythagorean view of 'knowledge is wonder'; the potential of mathematics as a route to higher orders of consciousness should be explored (Thompson, 1971; Ghyka, 1971; Musès, 1974; Young, 1976).

13) Mathematicians must become more aware and active in the area of social responsibility (Grothendieck, 1971).

14) The dangers of neo-Aristotelian either-or thinking, as exemplified in very common statements, particularly south of the border, such as 'America: Love it or Leave it', 'Live free or die', 'Are you with us or against us?' - should be revealed and pointed out as a case of inappropriate mathematical modelling. The 0-1 model is far too primitive for the issues to which it is often applied. The related issue of a misplaced emphasis on classification should also be considered (Korzybski, 1958; Hilton, 1975; Thompson, 1971; National Advisory, 1975).

15) Disciplinary specialization has been carried much too far in the academic world. There is a need to develop a system of education which, more than being interdisciplinary, is transdisciplinary. Such a system of integrative education can be based on a small number of fundamental organizing

concepts such as symme.ry, transformation, duality, continuity, and system (Margenau, 1972; Waddington, 1977). Whitehead was sensitive to the power of "big ideas" and the connection of mathematice to these ideas. "Nobody can be a good reasoner unless by constant practice he has realised the importance of getting hold of the big ideas and hanging on to them like grim death" (1949, p.87). "Mathematics well taught should be the most powerful instrument in gradually implanting this generality of idea. The essence of mathematics is perpetually to be discarding more special ideas in favour of general methods" (1949, p.60). Mathematical ideas would play a central role in integrative education. The concept of form would seem to be a particularly rich area (Thompson, 1969; Schwenk, 1965; Thom, 1975; Whyte, 1968).

As a final statement let me say that, in my opinion, if we should succeed in some of these tasks we might come somewhat closer to achieving the view of mathematics described some fourteen years ago by the distinguished mathematician and educator, Alexander Wittenberg, whose untimely death was a major blow to mathematics education in Canada. Wittenberg's statement (1963, p.1097), which I would like to close with, goes as follows:

> Mathematics exists, not as a defined entity in some logician's or philosopher's textbook, but first and foremost as a living reality, as a fact of life. We strive to understand it as we strive to understand all the other manifold aspects of our experience from physical nature to the nature of poetry. And we find ourselves faced with the same mixture of answered and unanswered questions, of insight and puzzlement, that is everywhere characteristic of the human situation.

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